

Problem 1: Early in the century an earthquake measured 8.0 on the Richter scale. In the same year, another earthquake was recorded that was six times stronger. What was the magnitude of the earthquake of the stronger earthquake?

Answer: 8.78 on the Richter Scale.

Solution: Convert the first sentence to an equivalent mathematical sentence or equation where

$$M_1 = \log \frac{I_1}{S} = 8.0$$

Convert the second sentence to an equivalent mathematical sentence or equation:

$$I_2 = 6 I_1$$

Convert the third sentence into an equivalent mathematical sentence or equation:

$$\begin{aligned} M_2 &= \log \frac{I_2}{S} \\ &= \log I_2 - \log S \end{aligned}$$

Replace I_2 with I_1

$$\begin{aligned} &= \log(6I_1) - \log S \\ &= \log 6 + \log I_1 - \log S \\ &= \log 6 + (\log I_1 - \log S) \\ &= \log 6 + \log \frac{I_1}{S} \\ &= \log 6 + 8.0 \\ &= 0.778151250384 + 8.0 \\ &= 8.778151250384 \\ M_2 &= 8.78 \end{aligned}$$

The intensity of the second earthquake was 8.78 on the Richter scale.

Let's check our answer.

$$8.78 = \log \frac{I_2}{S} = \log I_2 - \log S$$

$$8.0 = \log \frac{I_1}{S} = \log I_1 - \log S$$

$$8.78 - 8.0 = (\log I_2 - \log S) - (\log I_1 - \log S)$$

$$0.78 = \log I_2 - \log S - \log I_1 + \log S$$

$$0.78 = \log I_2 - \log I_1$$

$$0.78 = \log \frac{I_2}{I_1}$$

$$10^{0.78} = \frac{I_2}{I_1}$$

$$6.02559586074 \approx \frac{I_2}{I_1}$$